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(NASA-TM-89780) INVARIANCE CONDITIONS FOR LINEAR AUTOMATIC CONTROL SYSTEMS WITH PERTURBATIONS ASSIGNED STATISTICALLY (NASA)

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Invariance Conditions for Linear Automatic Control Systems with Perturbations Assigned Statistically

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An increase in the accuracy and high-speed of automatic regulation systems is a problem of practical importance. In recent years, a combination regulation principle, according to which ε terms corresponding to influences on the fundamental perturbations (i.e., from loading the object being regulated or from the program) are in the regulation law, has been used all the more widely for this purpose. The idea of selecting the regulation system circuit and parameters taking into account the continuous variation of the perturbations was first expressed in 1940 by Acad. V. S. Kulebakin. Conditions are given in his works [1,2,3] for the total elimination of the errors of linear systems when the perturbation is an arbitrary function of time (invariance for an arbitrary load) or a given function of time (invariance for a given load).

Let us assume that the automatic regulation system is described by a linear differential equation with constant coefficients

$$a_3(p)\varphi = - b_3(p)\,\lambda(t) + b_3''(p)\,\lambda''(t) \tag{1}$$

where φ is the deviation of the regulated quantity or the error for the servo-system; $\lambda(t)$ are the basic perturbations (loading for a stabilizing system or task for a following system); $a_3(p)$, $b_3(p)$ are polynomials in $p = \frac{d}{dt}$ of degree n and m, respectively; $b_3''(p)$ is a polynomial in $p = \frac{d}{dt}$; $\lambda''(t)$ are the additional external perturbations which are introduced artificially into the system from a forcing device.

The invariance conditions can then be written thus [4]:

In the absence of a forcing device

$$\lambda(t) = 0\,;\quad b_3(p) = 0\,;\quad b_3''(p) \neq 0 \quad\text{and}\quad \lambda(t) \neq 0 \quad\text{but}\quad b_3(p)\,\lambda(t) = 0$$

In the presence of a forcing device

$$- b_3(p)\,\lambda(t) + b_3''(p)\lambda''(t) = 0$$

Many automatic control systems experience the influence of perturbations which vary continuously and can only be given statistically. In connection with the development of statistical methods of investigating regulation systems [5,6], the question arises of the relation of these methods to the theory of combination regulation systems, in particular, of the expediency of adjusting systems in conformance with the invariance conditions. Are the invariance conditions effective for perturbations given statistically?

Let us analyze the most typical examples of synthesizing composite systems by using different forms of the invariance conditions.

EXAMPLE 1. Synthesis of a Composite System by Using the Second Form of the Invariance Conditions. Let us assume that the automatic regulation system is described by a linear differential equation with constant coefficients

$$a_3(p)\varphi = -b_3(p)\lambda(t) \tag{2}$$

The perturbation $\lambda(t)$ is given statistically. It is a stochastic function of time with variance D_λ and spectral density $S_\lambda(\omega)$. Let us agree that the function $\lambda(t)$ consists of analytic functions of time. An initial condition is also given: The system error is $\varphi(t) = 0$ at $t = 0$. It is required to find the coefficients of the polynomial $b_3(p)$ so that the system error will be a minimum.

The error spectral density is

$$S_\varphi(\omega) = \left[\frac{-b_3(p)}{a_3(p)}\right]^2 S_\lambda(\omega) \tag{3}$$

where $p = \frac{d}{dt}$; and the variance

$$D_\varphi = \int_{-\infty}^{+\infty} S_\varphi(\omega)\, d\omega \tag{4}$$

Therefore, the mean-square error of the system is

$$\varepsilon_\varphi = +\sqrt{D_\varphi} = +\sqrt{\int_{-\infty}^{+\infty} \left[\frac{-b_3(p)}{a_3(p)}\right]^2 S_\lambda(\omega)\, d\omega} \tag{5}$$

As long as we select $b_3(p)$ in accordance with the second form of the invariance conditions $b_3(p) = 0$, we obtain $D_\varphi = 0$ and $\varepsilon_\varphi = 0$. This means that φ can only be a constant. Using the initial condition $\varphi(t) = 0$ at $t = 0$, we easily find that the fixed system error becomes zero, $\varphi = 0$, under compliance with the invariance conditions. Hence, the invariance condition for arbitrary loading, $b_3(p) = 0$, is retained even for perturbations $\lambda(t)$ given statistically.

Taking into account that real systems are described by linear ordinary differential equations in a first approximation and also by a number of other limitations [4], the invariance conditions in all forms should be considered first as a directing means of decreasing (minimizing) error in every way.

EXAMPLE 2. Synthesis of Composite Systems Using the Fourth Form of the

Invariance Conditions. In practice, the invariance conditions in the second form, $b_3(p) = 0$, can be satisfied by using an appropriate choice of the coupling coefficients in terms of the basic perturbations and its derivative [4]. However, as a rule, the system must here have differentiators which give a first and second derivative in the perturbation. This causes no difficulty in electronic systems [7] but the second derivative cannot be obtained accurately enough in the absence of electronic amplifiers if considerable power is required at the differentiator output.

In this case, it is easy to reduce the system error by using a well-known forcing device [4] which additionally affects the system input $b_3''(p)\lambda''(t)$.

Let us consider the question of how to select the operator of the forcing device $b_3''(p)$ if the function is $\lambda''(t)$.

Let us assume that the automatic regulation system is described by the linear differential equation (1). The basic perturbing effect $\lambda(t)$ is given statistically. Let us assume that it is the sum

$$\lambda(t) = \lambda_1(t) + \lambda_2(t)$$

where $\lambda_1(t)$ is the usual, nonrandom function of time defined as the most probable mean value of the perturbing effect; its spectral density is $S_{\lambda_1}(\omega)$; $\lambda_2(t)$ is a random function of time with variance D_{λ_2} and spectral density $S_{\lambda_2}(\omega)$.

Also given is the initial condition: The system error is $\varphi(t) = 0$ at $t \leq 0$.

It is required to select the law of the variation of the effect of the forcing device $\lambda''(t)$ or its operator $b_3''(p)$ so that the system error would be a minimum.

The basic perturbing effect $\lambda(t)$ and the effect of the forcing device $\lambda''(t)$ are mutually independent. The correlation function is zero.

The error spectral density, under the effect of complex perturbations consisting of a random and a given function of time, is

$$S_\varphi(\omega) = \left[\frac{-b_3(p)}{a_3(p)}\right]^2 \left[S_{\lambda_1}(\omega) + S_{\lambda_2}(\omega)\right] + \left[\frac{b_3''(p)}{a_3(p)}\right]^2 S_{\lambda''}(\omega) \tag{6}$$

and the variance is

$$D_\varphi = \int_{-\infty}^{+\infty} S_\varphi(\omega)\, d\omega \tag{7}$$

The mean-square error is

$$(8) \quad \varepsilon_\varphi = D_\varphi = +\sqrt{\int_{-\infty}^{+\infty}\left\{\left[\frac{-b_3(p)}{a_3(p)}\right]^2\left[S_{\lambda_1}(\omega) + S_{\lambda_2}(\omega)\right] + \left[\frac{-b_3''(p)}{a_3(p)}\right]^2\left[S_{\lambda''}(\omega)\right]d\omega\right\}}$$

It will equal zero if the following invariance condition is satisfied:

$$(9) \qquad -S_{\lambda''}(\omega) = \left[\frac{-b_3(p)}{a_3(p)}\right]^2\left[S_{\lambda_1}(\omega) + S_{\lambda_2}(\omega)\right]$$

The error φ is a constant for $\varepsilon_\varphi = 0$.

Using the initial condition ($\varphi = 0$ at $t = 0$), we establish that the system error will be zero, $\varphi = 0$, if condition (9) is satisfied. Hence, we have obtained the desired invariance condition (9) which indicates a method of eliminating or, at least, of decreasing the error.

Let us turn attention to the fact that the polynomial $a_3(p)$ in the left side does not take part in condition (9), therefore, the possibility of eliminating or decreasing the error by using a forcing device is not dependent on the change in the system properties (its stability, say). It is possible to point out cases of different regulating systems in which it is possible to use the condition (9) to increase their accuracy. Let us visualize a system which includes a strip in which the influences are transmitted in the form of frequency combinations. Let it be assumed that an obstacle with spectral density $S_\lambda(\omega)$ is given in the whole strip. In order to eliminate the error that the obstacle causes, a forcing device (noise generator) with the spectral density $S_\lambda(\omega)$, which compensates the effect of the obstacle, can be used.

However, systems in which the effects of the forcing device $\lambda''(t)$ are realized as a statistical, random function of time are very few. Most often, the effects of the forcing device $\lambda''(t)$ can be realized in the systems as the usual, nonrandom function of time. Consequently, the further transformation of the condition (9) (in contrast to case 1 where complete error elimination is achieved at first glance) affords the possibility of determining the system adjustment which corresponds just to a change in the error, or more exactly, to elimination of the average component of the error which is ε .

The error component which arises under the influence of random perturbations $\lambda_2(t)$ is retained. Only the component which arises under the action of $\lambda_1(t)$ is eliminated.

The spectral density is related to the square of the mathematical expectation by using the Fourier transformation. Hence,

$$M_{\lambda''} = \left[\frac{b_3(p)}{b_3''(p)}\right](M_{\lambda_1} + M_{\lambda_2}) \tag{10}$$

The mathematical expectation of a nonrandom function of time equals

$$M_{\lambda''} = \lambda''(t) \tag{11}$$

$$M_{\lambda_1} = \lambda_1(t) \tag{12}$$

and the mathematical expectation of a stationary random function is zero

$$M_{\lambda_2} = 0 \tag{13}$$

Thus, we finally obtain

$$\lambda''(t) = \left[\frac{b_3(p)}{b_3''(p)}\right]\lambda_1(t) \tag{14}$$

Condition (14) permits the forcing apparatus $\lambda''(t)$ and the operator $b''(t)$ to be selected so that the error component which arises because of $\lambda_1(t)$ would be eliminated completely.

Under the conditions of the problem being considered, the error will be decreased sharply by the magnitude of the average component. The average component of the effect $\lambda_1(t)$ does not cause any error to appear.

We obtained the invariance condition for perturbations usually given as nonrandom functions of time, earlier [4], in the so-called fourth form

$$\lambda''(t) = \frac{b_3(p)}{b_3''(p)}\lambda(t) \tag{15}$$

When the perturbations are given thus

$$\lambda(t) = M_\lambda$$

then the result (15), which we obtained earlier, corresponds completely to condition (14).

Hence, we have shown that the invariance conditions for linear systems are real for any given, including statistically, perturbations. The form of the invariance conditions is almost unchanged.

Inst. of Electrical Engineering, Ukr. SSR March 10, 1957

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